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# Cosmic Numbers and Quantum Theory

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The cosmic numbers are considered, with emphasis on the relation $N \sim \rho^2$. (Here $N$ is the number of nucleons in the universe, and $\rho$, its radius in atomic units.) This relation is interpreted in terms of a quantum mechanical model.

## 1. INTRODUCTION

In the present note we continue the discussion of Tarski (1984, Appendix) on the relations among the cosmic numbers $N$, $\rho$, and $G$. Here N is the number of nucleons in the universe, each having (approximately) the mass m, $\rho$ is the radius of the universe, and G, the gravitational constant. For $\rho$ and $G$ we use the atomic units, where $c = \hbar = m = 1$. We then have the estimates

$$N \sim 10^{80}, \qquad \rho \sim 10^{40}, \qquad G \sim 10^{-40} \tag{1}$$

We should now like to recall some of the observations made in Tarski (1984).

Let us set $M = mN$; we see that

$$G \sim \rho / M \tag{2}$$

This relation presupposes $c = 1$, but it does not depend on $\hbar$, nor on $m$ and $N$ separately, and so it involves only classical parameters. This relation has been encountered in a number of discussions. It can be considered as an expression of Mach's principle, insofar as it determines $G$, and hence inertia, in terms of mass and geometry.

Relations among the cosmic numbers independent of (2) will necessarily depend on setting $\hbar = 1$, and they also deserve a serious study. In Tarski (1984) we pointed in particular to

$$N \sim \rho^2 \tag{3}$$

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If we use arbitrary units restricted by $c = 1$ only, then this can be rewritten

$$\hbar / m \sim \rho / N^{1/2} \tag{4}$$

This relation could be considered, in analogy to (2), as an expression of a quantum-theoretic extension of Mach's principle.

Eddington (1953) offers a derivation of (3) in the context of fluctuations within classical statistical physics, but his analysis comes close to quantum-theoretic notions. In this note we will interpret (3), (4) in terms of a quantum-mechanical example, which exploits the ideas of Eddington, but in which the role of $\hbar$ will be in the foreground.

It is clear that such an example must go beyond the standard framework of quantum mechanics, since in this framework $\hbar/m$ is arbitrary, and so cannot be determined. However, other attempts to set up quantum mechanics for the universe as a whole (e.g. DeWitt, 1967; Bell, 1975) have similarly indicated the need for modifying this framework, e.g., in view of the nonexistence of an outside observer.

We emphasize that our example (in Section 3) should only be considered as a suggestion, one that could be developed. However, we felt that a detailed analysis could perhaps be best postponed until the physical ingredients are better understood.

It may be appropriate to mention here another reference relating to (3), namely, Kilmister and Tupper (1962). This book contains a variant of Eddington's derivation of (3), remarks bearing on the empirical relevance of this relation for atomic and nuclear physics, and references to previous articles of the authors.

## 2. REMARKS ON A THEORY OF DIRECT PARTICLE INTERACTIONS

The formulation of gravitation theory due to Hoyle and Narlikar (1964, 1974) provides a useful background for our subsequent example. These authors define the theory by the following action, which is given in terms of scalar massless Green's functions $\tilde{G}$ integrated along world lines:

$$S = \tfrac{1}{2}\lambda \sum_{u \neq v} \iint du\, dv\, \tilde{G}(U, V) \tag{5}$$

The sum extends over all pairs of particles, and $\lambda$ is a coupling constant. At this stage there is no mass parameter (however, mass appears in later calculations), and similarly $\hbar$ does not enter. Cf. also the review of Raine (1981).

The underlying space-time is pseudo-Riemannian, but for the case of small deviations from Minkowski space, the curvature effects can be replaced by a gravitational field. We then retain the usual dynamical quantities $p_x$, etc.

Assume now $N+1$ particles, one of which is singled out and labeled "$A$." Consider the associated generator $i^{-1}\partial_x^{(A)}$. According to (5), this generator acts on $N$ Green's functions. To each $\tilde{G}(A, j)$ there corresponds a distribution of momenta $p_x^{(j)}$. Let us suppose that we are in a situation where the central limit theorem is applicable, so that we may write

$$-\sum p_x^{(j)} \sim N^{1/2} p_x^{(A)} \tag{6a}$$

Thus $i^{-1}\partial_x^{(A)} \leftrightarrow N^{1/2} p_x^{(A)}$, and

$$\hbar \propto N^{-1/2} \tag{6b}$$

We see in (6b) the main feature of (4). We emphasize that we obtained this proportionality with the help of the central limit theorem, and it seems that it can be obtained only in this way. This theorem was also exploited by Eddington (1953) for analysis of fluctuations.

Note that in the foregoing

$$\partial_x^{(U)} \tilde{G}(U, V) = -\partial_x^{(V)} \tilde{G}(U, V) \tag{7}$$

We might therefore say that the dependence of $\hbar$ on $N$ arises by associating the operators to pairs to particles rather than to individual particles.

We remark, moreover, that it should be possible to transcribe (5) to a spherical background space, with which we will be dealing next. [Cf. Tarski (1984), Section 5.]

## 3. QUANTUM PARTICLES ON $S^3(\rho)$

The example that we are about to describe should be regarded as independent of the previous, which was based on the action (5), even though there are evident points of contact.

We take $N+1$ quantum particles on $S^3(\rho)$, the three-sphere of radius $\rho$. The particles are assumed to be interacting, e.g., gravitationally, and the dynamics is assumed to depend only on relative coordinates (see below). In addition, we assume that the distributions of quantum-mechanical variables are such that the central limit theorem applies.

We single out a particle which we label "$A$," and we consider the azimuthal angle $\varphi$ on $S^3(\rho)$ and the generator $i^{-1}\partial_\varphi^{(A)}$. Then this generator has to affect all other particles as well. We express the result of its action as follows:

$$-\sum p_\varphi^{(j)} \sim -N^{1/2} \bar{p}_\varphi \sim N^{1/2} p_\varphi^{(A)} \tag{8}$$

Then we obtain, as in (6a, b), the dependence $\hbar \propto N^{-1/2}$.

(Note that the hypothesis of relative coordinates may require some corrections when angular variables are used. Note also that the gradual reduction of $p_\varphi$ from maximum at the equator to zero at the poles has to be taken into account. However, these details should not change the above order-of-magnitude estimates.)

To determine the constant of proportionality for $\hbar \propto N^{-1/2}$, consider the case $N = 2$, which should have the characteristics of the hydrogen atom problem. In particular, the system should extend over (roughly) a distance $D$ satisfying

$$D \sim \hbar/m \lesssim \rho \tag{9}$$

The natural choice, but not a compelling one, is

$$\hbar/m \sim \rho \qquad \text{(for small } N) \tag{10}$$

If we accept (10) as well as $\hbar \propto N^{-1/2}$, then we immediately obtain (4), as desired.

The relation (10) deserves a comment. It figures here as a new hypothesis. Its interpretation could be, e.g., that for small $N$ the size of the universe is determined primarily by the matter quanta, since there would not be enough radiation to inflate it, so to speak.

One final comment. If the particles are noninteracting, or if $N = 1$, then the wave function for each particle should extend over the whole space. By admitting interactions we reduce $\hbar/m$, and hence the extent of the wave functions. This is analogous to the usual "reduction of wave packets" by interactions.

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